Even- and Odd-Mode Waves for Nonsymmetrical Coupled Lines in Nonhomogeneous Media

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Abstract—A simple analysis of the eigenvectors, representing the fundamental uncoupled wave modes of a pair of nonsymmetrical coupled lines in nonhomogeneous medium, proves that these two modes reduce, under a given condition, to an even mode with equal voltage magnitudes and an odd mode with equal current magnitudes and opposite polarities. The condition, which may be cast in many representative forms, is called "congruence condition" and may be formulated simply, for a nonhomogeneous dielectric medium, by saying that the ratio of the per-unit-length conductor-to-ground capacitances must be the same in the empty and in the filled structure. The essential interest of congruence lies in the drastic simplification it introduces in the expressions of the eigenvalues and of the mode velocities and in the expressions of the mode admittances and impedances for the two coupled lines. Because of this simplification, a straightforward matrix derivation may be written to obtain closed-form expressions of the entries of the 4 × 4 Y-, Z-, and S-parameter matrices of the coupled-line 4-port. The simplicity of the definition of the fundamental modes in congruent structures introduces great conceptual clarity in the description of coupled-wave propagation. Experimental evidence is presented which proves the physical existence of the even-mode wave and of the redefined odd-mode wave in suspended-substrate broadside-coupled striplines. Practical structures of this type are very closely congruent.

I. INTRODUCTION

THE propagation of sinusoidal waves on parallel coupled lines has been described in various ways by different authors at different times.

Historically, the even- and odd-mode method was first applied to the case of symmetrical lines in a homogeneous medium [1]. The same method was later extended to the case of symmetrical lines in a nonhomogeneous medium [2] and the case of nonsymmetrical lines in a homogeneous medium [3], [4]. In this latter case a rather confusing alternative was given in the definition of the modes, depending on whether the line voltages or the line currents were considered [5]. Both these definitions appear valid in the case of a homogeneous medium while it can be shown that neither of the two applies to the general case of nonsymmetrical lines in a nonhomogeneous medium [6].

Attempts have been made to treat this general case in terms of so-called coupled modes [7], [8]. However, by far the most concise formulation of the problem was given in terms of a matrix analysis of the fundamental or uncoupled modes [9]–[11]. This method is very general and applies to the case of multiconductor transmission lines with any number of conductors all mutually coupled. It is the purpose of this paper to show that a very interesting and simple condition exists in the case of two nonsymmetrical parallel coupled lines in a nonhomogeneous medium which reduces the fundamental modes to a voltage even mode and a current odd mode. Because of this reduction, great simplification is introduced in the expressions of the mode velocities and of the mode admittances and impedances for the two lines.

This simplification makes the derivation of the 4 × 4 Y-, Z-, and S-matrices feasible in closed form. Also the condition, which we call "congruence," introduces great conceptual clarity in the normal mode description of wave propagation.

Experimental confirmation has been obtained of the physical existence of the reduced fundamental modes. This work is described in the last section of this paper.

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II. THE FUNDAMENTAL MODES OF GENERALIZED COUPLED LINES

Multiconductor coupled transmission lines, including those with a nonhomogeneous medium, are characterized by their symmetric inductance matrix $|L|$ and their symmetric capacitance matrix $|C|$. The order n of these matrices is given by the number of conductors in the system, and their entries L_{ij}, C_{ij} are the per-unit-length self- or mutual inductances and capacitances of the various conductors.

Sinusoidal waves propagating on the lines may be described by a voltage vector $|V|$ and a current vector $|I|$ both of order n. These vectors may be expressed as linear combinations of n voltage eigenvectors $|V|_i$ and, respectively, n current eigenvectors $|I|_i$ defined by the two simultaneous matrix equations [11] ($i = 1,2\cdots,n$)

$$|V|_i = v_i \cdot |L| \cdot |I|_i \tag{1}$$

$$|I|_i = v_i \cdot |C| \cdot |V|_i \tag{2}$$

where the mode velocity v_i and the voltage and current eigenvectors $|V|_i$ and $|I|_i$ characterize a fundamental or uncoupled mode of propagation.

By eliminating either the eigenvector $|I|_i$ from (1) or the eigenvector $|V|_i$ from (2), two eigenvalue equations for $|V|_i$ and, respectively, $|I|_i$, are obtained

$$|LC| \cdot |V|_i = \frac{1}{v_i^2} |V|_i \tag{3}$$

$$|CL| \cdot |I|_i = \frac{1}{v_i^2}|I|_i \qquad (4)$$

where $|LC| = |L| \cdot |C|$ and $|CL| = |C| \cdot |L|$ are the direct and inverse matrix products of matrices $|L|$ and $|C|$.

In the case of a pair of lossless nonsymmetrical parallel coupled lines, including the case of a nonhomogeneous medium, these matrices may be written as

$$|L| = \begin{vmatrix} L_1 & L_m \\ L_m & L_2 \end{vmatrix} = \begin{vmatrix} L_a & M \\ M & L_b \end{vmatrix} \qquad (5)$$

$$|C| = \begin{vmatrix} C_1 & -C_m \\ -C_m & C_2 \end{vmatrix} = \begin{vmatrix} C_a + C_{ab} & -C_{ab} \\ -C_{ab} & C_b + C_{ab} \end{vmatrix} \qquad (6)$$

and their products assume the form

$$|LC| = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \qquad (7)$$

$$|CL| = |LC|^T = \begin{vmatrix} A & C \\ B & D \end{vmatrix} \qquad (8)$$

where

$$A = L_1C_1 - L_mC_m \qquad (9)$$

$$B = -L_1C_m + L_mC_2 \qquad (10)$$

$$C = L_mC_1 - L_2C_m \qquad (11)$$

$$D = L_2C_2 - L_mC_m. \qquad (12)$$

By substituting the products (7) and (8) in the eigenvalue equations (3) and (4) and expressing the eigenvectors $|V|_i$ and $|I|_i$ in normalized form, we obtain the matrix equations

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} \cdot \begin{vmatrix} 1 \\ \alpha_i \end{vmatrix} \cdot V = \frac{1}{v_i^2}\begin{vmatrix} 1 \\ \alpha_i \end{vmatrix} V, \qquad i = 1,2 \qquad (3')$$

$$\begin{vmatrix} A & C \\ B & D \end{vmatrix} \cdot \begin{vmatrix} 1 \\ \beta_i \end{vmatrix} \cdot I = \frac{1}{v_i^2}\begin{vmatrix} 1 \\ \beta_i \end{vmatrix} I, \qquad i = 1,2 \qquad (4')$$

which may be solved for the voltage and current mode numbers α_i and β_i

$$\alpha_i^2 + \frac{A-D}{B}\alpha_i - \frac{C}{B} = 0 \qquad (13)$$

$$\alpha_i = -\frac{A-D}{2B} \pm \frac{1}{2B}[(A-D)^2 + 4BC]^{1/2} \qquad (13')$$

$$\beta_i^2 + \frac{A-D}{C}\beta_i - \frac{B}{C} = 0 \qquad (14)$$

$$\beta_i = -\frac{A-D}{2C} \pm \frac{1}{2C}[(A-D)^2 + 4BC]^{1/2} \qquad (14')$$

and for the eigenvalues $1/v_i^2$ by solving the characteristic equation of $|LC|$ or $|CL|$[1] [12]

$$\left(\frac{1}{v_i^2}\right)^2 - (A+D)\frac{1}{v_i^2} + (AD - BC) = 0 \qquad (15)$$

we then obtain the two values

$$\frac{1}{v_i^2} = \frac{A+D}{2} \pm \tfrac{1}{2}[(A-D)^2 + 4BC]^{1/2} \qquad (15')$$

which define the velocities of the fundamental modes. It is easy to express the mode impedances and admittances as functions of the per-unit-length inductances and capacitances, of the mode velocities and of the mode numbers α_i and β_i. To do this we just rewrite (1) and (2) by substituting (5) and (6) and expressing the eigenvectors in normalized form ($i = 1,2$)

$$\begin{vmatrix} 1 \\ \alpha_i \end{vmatrix} V = v_i \begin{vmatrix} L_1 & L_m \\ L_m & L_2 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ \beta_i \end{vmatrix} I \qquad (1')$$

$$\begin{vmatrix} 1 \\ \beta_i \end{vmatrix} I = v_i \begin{vmatrix} C_1 & -C_m \\ -C_m & C_2 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ \alpha_i \end{vmatrix} V \qquad (2')$$

or explicitly ($i = 1,2$)

$$V = v_i(L_1 + \beta_iL_m)I \qquad (16)$$

$$\alpha_iV = v_i(L_m + \beta_iL_2)I \qquad (17)$$

$$I = v_i(C_1 - \alpha_iC_m)V \qquad (18)$$

$$\beta_iI = v_i(-C_m + \alpha_iC_2)V. \qquad (19)$$

As a consequence, the mode impedances and admittances of the two lines a and b for mode i are ($i = 1,2$)

$$Z_{0i}^a = \frac{V}{I} = v_i(L_1 + \beta_iL_m) \qquad (16')$$

$$Z_{0i}^b = \frac{\alpha_iV}{\beta_iI} = v_i\left(L_2 + \frac{L_m}{\beta_i}\right) \qquad (17')$$

$$Y_{0i}^a = \frac{I}{V} = v_i(C_1 - \alpha_iC_m) \qquad (18')$$

$$Y_{0i}^b = \frac{\beta_iI}{\alpha_iV} = v_i\left(C_2 - \frac{C_m}{\alpha_i}\right). \qquad (19')$$

Thus we obtain four impedances and four admittances, one for each mode and each line. Notice that

$$\frac{Y_{0i}^a}{Y_{0i}^b} = \frac{Z_{0i}^b}{Z_{0i}^a} = \frac{\alpha_i}{\beta_i} = \frac{C}{B}, \qquad i = 1,2.$$

It is clear that in the general case the mode numbers α_i and β_i, the mode velocities v_i, and the mode impedances and admittances are all quite complex irrational functions

[1] The coefficients of this equation are $-\mathrm{Tr}\,|LC|$ and $\mathrm{Det}\,|LC|$.

of the per-unit-length inductances L_a, L_b, M, and capacitances C_a, C_b, C_{ab}. However, α_i, β_i, and v_i can be proved to be always real. As a consequence, the mode impedances and admittances will be real too, for lossless lines.

The mode numbers α_i and β_i may have either positive or negative sign and may have the same or opposite signs. The mode velocities v_i obtained from (15′) always come in pairs of opposite roots, relating to the two directions of possible mode propagation on the two coupled lines.

III. THE CONGRUENCE CONDITION AND ITS CONSEQUENCES

It is easy to see that great simplification is introduced in the expressions of the mode numbers (13′), (14′), the eigenvalues (15′), and mode impedances and admittances (16′)–(19′) if, in (7) and (8)

$$A + B = C + D \tag{20}$$

or

$$A - D = C - B. \tag{20'}$$

In this case, which we call congruence, the mode numbers α_i and β_i reduce to

$$\alpha_i = -\frac{C-B}{2B} \pm \frac{1}{2B}[(C-B)^2 + 4BC]^{1/2}$$
$$= \tfrac{1}{2}[(1 - C/B) \pm (1 + C/B)]$$
$$= \begin{cases} 1 & \text{for } i = 1\ (+\text{ sign}) \\ -C/B & \text{for } i = 2\ (-\text{ sign}) \end{cases} \tag{13''}$$

$$\beta_i = -\frac{C-B}{2C} \pm \frac{1}{2C}[(C-B)^2 + 4BC]^{1/2}$$
$$= \tfrac{1}{2}[B/C - 1) \pm (B/C + 1)]$$
$$= \begin{cases} B/C & \text{for } i = 1\ (+\text{ sign}) \\ -1 & \text{for } i = 2\ (-\text{ sign}). \end{cases} \tag{14''}$$

In conclusion, in this case the matrices of the normalized voltage and current eigenvectors reduce to

$$M_V = \begin{vmatrix} 1 & 1 \\ 1 & -C/B \end{vmatrix} \tag{21}$$

$$M_I = \begin{vmatrix} 1 & 1 \\ B/C & -1 \end{vmatrix}. \tag{22}$$

From the first column of M_V we see that the mode $i = 1$ is a voltage even mode having equal voltages on the two lines. From the second column of M_I we see that the mode $i = 2$ is a current odd mode having currents of equal magnitude and opposite sign on the two lines.

It is then justified to call these modes the even mode and the redefined odd mode of the congruent coupled lines. A physical interpretation of the condition (20) may be obtained by substituting the expressions (9)–(12) of A, B, C, and D

$$L_1C_1 - L_mC_m - L_1C_m + L_mC_2$$
$$= L_mC_1 - L_2C_m + L_2C_2 - L_mC_m \tag{23}$$

$$L_1(C_1 - C_m) + L_m(C_2 - C_m)$$
$$= L_2(C_2 - C_m) + L_m(C_1 - C_m) \tag{23'}$$

$$\frac{C_1 - C_m}{C_2 - C_m} = \frac{L_2 - L_m}{L_1 - L_m} \tag{23''}$$

$$\frac{C_a}{C_b} = \frac{L_b - M}{L_a - M}. \tag{23'''}$$

We see from (23‴) that a pair of coupled lines may, while being congruent, be nonsymmetrical ($C_a \neq C_b$, $L_a \neq L_b$) and the medium may be nonhomogeneous causing the mode velocities $v_1 = v_e$ for the even mode and $v_2 = v_o$ for the redefined odd mode to be different.

In particular, if the medium is just a nonhomogeneous dielectric with $\mu_r = 1$, then the parameters L_a, L_b, and M will have the same values in the given structure and in the so-called empty structure, which is obtained by removing the dielectric while leaving in place the conductors.

In this connection it is interesting to notice that the congruence condition (20) is intrinsically satisfied in the empty structure where

$$v_1 = v_2 = c = \text{velocity of light in air (or vacuum)}.$$

Indeed, in this case the product matrices $|LC|$ and $|CL|$ are diagonal matrices with

$$B = C = 0 \tag{24}$$

and

$$A = D = 1/c^2. \tag{25}$$

This means that the condition (23‴), equivalent to (20), will always be satisfied by the per-unit-length line-to-ground capacitances C_{aA},C_{bA} of the empty structure and by its per-unit-length inductance parameters L_a, L_b, and M.

In conclusion, if a pair of nonsymmetrical coupled lines with nonhomogeneous dielectric is congruent and we call C_{aD},C_{bD} its per-unit-length line-to-ground capacitances (with the dielectric in place), we have

$$\frac{C_{aD}}{C_{bD}} = \frac{C_{aA}}{C_{bA}} = \frac{L_b - M}{L_a - M}. \tag{26}$$

So the congruence condition may be expressed by saying that the introduction of the nonhomogeneous dielectric in the empty structure must not affect the value of the ratio of the per-unit-length line-to-ground capacitances.

The effect of the nonhomogeneous dielectric may be described by introducing the three equivalent dielectric constants ϵ_a, ϵ_b, and ϵ_m defined by

$$C_{aD} = \epsilon_a C_{aA} \tag{27}$$

$$C_{bD} = \epsilon_b C_{bA} \tag{28}$$

$$C_{abD} = \epsilon_m C_{abA}. \tag{29}$$

Then congruence is obtained if $\epsilon_a = \epsilon_b$.

It can be proved that for a nonhomogeneous medium having at the same time dielectric and magnetic properties, the latter being described by the three equivalent relative permeabilities μ_a, μ_b, and μ_m, congruence is obtained if

$$\mu_a\epsilon_a - \mu_b\epsilon_b = (\mu_m - \mu_a)\epsilon_a \frac{C_{abA}}{C_{bA}} - (\mu_m - \mu_b)\epsilon_b \frac{C_{abA}}{C_{aA}}. \tag{30}$$

For a pair of congruent, nonsymmetrical coupled lines with nonhomogeneous medium, the only nonunity mode numbers are

$$\alpha_2 = \alpha_o = -\frac{C}{B} \tag{31}$$

and

$$\beta_1 = \beta_e = \frac{B}{C}. \tag{32}$$

Under conditions of congruence the basic expression

$$\frac{C}{B} = \frac{L_m C_1 - L_2 C_m}{L_m C_2 - L_1 C_m} \tag{33}$$

also becomes greatly simplified and acquires a relevant physical meaning. This may be seen by rewriting (23′) as

$$L_1(C_1 - C_m) - L_2(C_2 - C_m) = L_m(C_1 - C_m) - L_m(C_2 - C_m). \tag{23''''}$$

By multiplying both sides by C_m and adding $L_m C_1 C_2$, this may be rearranged in the form

$$(L_m C_1 - L_2 C_m)(C_2 - C_m) = (L_m C_2 - L_1 C_m)(C_1 - C_m). \tag{23'''''}$$

So that in conclusion

$$\frac{C}{B} = \frac{L_m C_1 - L_2 C_m}{L_m C_2 - L_1 C_m} = \frac{C_1 - C_m}{C_2 - C_m} = \frac{C_a}{C_b}. \tag{33'}$$

We see then that in the case of congruent lines the even-mode currents are simply in the same mutual ratio as the per-unit-length line-to-ground capacitances. Also the odd-mode voltage magnitudes are in the reciprocal ratio of these capacitances

$$\alpha_o = -\frac{C_a}{C_b} \tag{31'}$$

$$\beta_e = \frac{C_b}{C_a}. \tag{32'}$$

The congruence condition (20) also introduces great simplification in the expressions of the eigenvalues as given by (15′). Indeed these expressions reduce to

$$\frac{1}{v_i^2} = \frac{A+D}{2} \pm \tfrac{1}{2}[(C+B)^2]^{1/2} = \tfrac{1}{2}[(A+D) \pm (C+B)]$$
$$= \begin{cases} \tfrac{1}{2}[(A+B) + (C+D)] = A+B = C+D & \text{for } i=1 \\ \tfrac{1}{2}[(A-C) + (D-B)] = A-C = D-B & \text{for } i=2. \end{cases} \tag{15''}$$

As a consequence, very simple expressions of the mode velocities may be written as functions of the line parameters

$$v_1 = v_e = \frac{1}{(A+B)^{1/2}} = \frac{1}{(C+D)^{1/2}} = \frac{1}{[L_1(C_1 - C_m) + L_m(C_2 - C_m)]^{1/2}} = \frac{1}{[L_2(C_2 - C_m) + L_m(C_1 - C_m)]^{1/2}} \tag{34}$$

$$v_2 = v_o = \frac{1}{(A-C)^{1/2}} = \frac{1}{(D-B)^{1/2}} = \frac{1}{[(L_1 - L_m)C_1 + (L_2 - L_m)C_m]^{1/2}} = \frac{1}{[(L_2 - L_m)C_2 + (L_1 - L_m)C_m]^{1/2}}. \tag{35}$$

Finally, from the expressions (16′)–(19′) we obtain, by substituting the appropriate values of the mode numbers α_i and β_i, the following expressions of the even- and odd-mode impedances and admittances:

$$Z_{0e}^a = v_e\left(L_1 + \frac{C_b}{C_a}L_m\right) \tag{36}$$

$$Z_{0e}^b = v_e\left(L_2 + \frac{C_a}{C_b}L_m\right) \tag{37}$$

$$Z_{0o}^a = v_o(L_1 - L_m) \tag{38}$$

$$Z_{0o}^b = v_o(L_2 - L_m) \tag{39}$$

$$Y_{0e}^a = v_e(C_1 - C_m) = v_e C_a \tag{40}$$

$$Y_{0e}^b = v_e(C_2 - C_m) = v_e C_b \tag{41}$$

$$Y_{0o}^a = v_o\left(C_1 + \frac{C_a}{C_b}C_m\right) \tag{42}$$

$$Y_{0o}^b = v_o\left(C_2 + \frac{C_b}{C_a}C_m\right). \tag{43}$$

By substituting $C_1 = C_a + C_{ab}$, $C_2 = C_b + C_{ab}$, and $C_m = C_{ab}$ in (42) and (43) and taking the reciprocals, the odd-mode impedances may be cast in a more familiar form

[5, p. 193]

$$Z_{0o}{}^{a} = \frac{C_b}{v_o(C_aC_b + C_aC_{ab} + C_bC_{ab})} \tag{38'}$$

$$Z_{0o}{}^{b} = \frac{C_a}{v_o(C_aC_b + C_aC_{ab} + C_bC_{ab})}. \tag{39'}$$

It is clear from all the expressions found for the congruent case that the ratio C_a/C_b of the per-unit-length line-to-ground capacitances plays a very important role here. Historically [6], the symbol R_3 was used to specify this ratio as follows:

$$R_1 = \frac{C_{ab}}{C_a} \quad R_2 = \frac{C_{ab}}{C_b} \quad R_3 = \frac{C_a}{C_b} \quad R_4 = \frac{C_{ab}}{(C_aC_b)^{1/2}}.$$

Among these ratios only R_3 and R_4 actually play roles of interest in the relevant expressions of the congruent coupled lines.

The even and odd modes of congruent lines have been proven to satisfy the coupled-mode equations [13].

IV. The Derivation of the Y-, Z-, and S-Matrices

In the coupled-line 4-port of Fig. 1 where line a connects ports 1 and 2 and line b connects ports 3 and 4, we consider the contiguous ports 1 and 3 input ports, and the contiguous ports 2 and 4 output ports.

In the context of the new mode definition we can write the four port currents I_1, I_2, I_3, and I_4 and the four port voltages V_1, V_2, V_3, and V_4 as follows:

$$I_1 = I_{1e} + I_{1o} \tag{44}$$

$$I_2 = I_{2e} + I_{2o} \tag{45}$$

$$I_3 = I_{3e} - I_{1o} \tag{46}$$

$$I_4 = I_{4e} - I_{2o} \tag{47}$$

$$V_1 = V_{1e} + V_{1o} \tag{48}$$

$$V_2 = V_{2e} + V_{2o} \tag{49}$$

$$V_3 = V_{1e} + V_{3o} \tag{50}$$

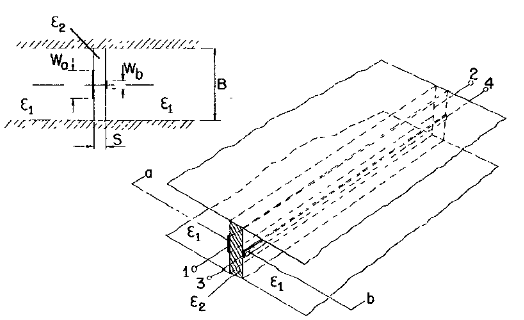


Fig. 1. Typical nonsymmetrical asynchronous coupled-line 4-port. Nonhomogeneous dielectric propagation medium is assumed. The cross-section geometry of the test fixture for wave-mode propagation experiments is shown in the insert.

$$V_4 = V_{2e} + V_{4o}. \tag{51}$$

In (44)–(47) the even-mode current components are then mutually related by the simple proportion

$$\frac{I_{1e}}{I_{3e}} = \frac{I_{2e}}{I_{4e}} = \frac{C_a}{C_b} = R_3. \tag{52}$$

Similarly, (48)–(51) the odd-mode voltage components are mutually related by the proportion

$$\frac{V_{1o}}{V_{3o}} = \frac{V_{2o}}{V_{4o}} = -\frac{C_b}{C_a} = -\frac{1}{R_3}. \tag{53}$$

As a consequence the eight expressions (44)–(51) can be rewritten as

$$I_1 = I_{1e} + I_{1o} \tag{44'}$$

$$I_2 = I_{2e} + I_{2o} \tag{45'}$$

$$I_3 = \frac{I_{1e}}{R_3} - I_{1o} \tag{46'}$$

$$I_4 = \frac{I_{2e}}{R_3} - I_{2o} \tag{47'}$$

$$V_1 = V_{1e} + V_{1o} \tag{48'}$$

$$V_2 = V_{2e} + V_{2o} \tag{49'}$$

$$V_3 = V_{1e} - R_3V_{1o} \tag{50'}$$

$$V_4 = V_{2e} - R_3V_{2o}. \tag{51'}$$

In these equations the even-mode current and voltage components at the input and output ports are mutually related by the two 2-port even-mode Y-matrices $|Y_E{}^a|$ and $|Y_E{}^b|$, which are different, because of line inequality

$$\begin{vmatrix} I_{1e} \\ \\ I_{2e} \end{vmatrix} = |Y_E{}^a| \cdot \begin{vmatrix} V_{1e} \\ \\ V_{2e} \end{vmatrix} \tag{54}$$

$$\begin{vmatrix} \dfrac{I_{1e}}{R_3} \\ \\ \dfrac{I_{2e}}{R_3} \end{vmatrix} = |Y_E{}^b| \cdot \begin{vmatrix} V_{1e} \\ \\ V_{2e} \end{vmatrix}. \tag{55}$$

Similarly, the odd-mode voltage and current components at the input and output ports are mutually related by the two different 2-port odd-mode Z-matrices $|Z_O{}^a|$ and $|Z_O{}^b|$

$$\begin{vmatrix} V_{1o} \\ \\ V_{2o} \end{vmatrix} = |Z_O{}^a| \cdot \begin{vmatrix} I_{1o} \\ \\ I_{2o} \end{vmatrix} \tag{56}$$

$$\begin{vmatrix} -R_3 V_{1o} \\ \\ -R_3 V_{2o} \end{vmatrix} = | Z_O{}^b | \cdot \begin{vmatrix} -I_{1o} \\ \\ -I_{2o} \end{vmatrix}. \tag{57}$$

However, by performing an operation of matrix inversion, which implies a network-parameter transformation from Z to Y, these relations can be rewritten in the form

$$\begin{vmatrix} I_{1o} \\ \\ I_{2o} \end{vmatrix} = | Z_O{}^a |^{-1} \cdot \begin{vmatrix} V_{1o} \\ \\ V_{2o} \end{vmatrix} \tag{58}$$

$$\begin{vmatrix} -I_{1o} \\ \\ -I_{2o} \end{vmatrix} = | Z_O{}^b |^{-1} \cdot \begin{vmatrix} -R_3 V_{1o} \\ \\ -R_3 V_{2o} \end{vmatrix}. \tag{59}$$

In (54) and (55) the even-mode 2-port Y-matrices for lines a and b are given by

$$| Y_E{}^x | = \begin{vmatrix} -j \dfrac{Y_{0e}{}^x}{\tan \theta_e} & j \dfrac{Y_{0e}{}^x}{\sin \theta_e} \\ \\ j \dfrac{Y_{0e}{}^x}{\sin \theta_e} & -j \dfrac{Y_{0e}{}^x}{\tan \theta_e} \end{vmatrix}, \qquad x = a,b. \tag{60}$$

At the same time, the two odd-mode 2-port Z-matrices of (58) and (59) are given by

$$| Z_O{}^x | = \begin{vmatrix} -j \dfrac{Z_{0o}{}^x}{\tan \theta_o} & -j \dfrac{Z_{0o}{}^x}{\sin \theta_o} \\ \\ -j \dfrac{Z_{0o}{}^x}{\sin \theta_o} & -j \dfrac{Z_{0o}{}^x}{\tan \theta_o} \end{vmatrix}, \qquad x = a,b. \tag{61}$$

In the given expressions (60) and (61) θ_e and θ_o are the line electrical lengths for, respectively, the even and odd mode as defined by

$$\theta_e = \beta_e l = \frac{2\pi}{\lambda_e} l = \frac{\omega}{v_e} l \tag{62}$$

$$\theta_o = \beta_o l = \frac{2\pi}{\lambda_o} l = \frac{\omega}{v_o} l \tag{63}$$

while the even-mode admittances $Y_{0e}{}^a, Y_{0e}{}^b$ and the odd-mode impedances $Z_{0o}{}^a, Z_{0o}{}^b$ are defined by (40), (41), (38′), and (39′).

By solving (48′) and (50′) for V_{1e} and V_{1o}, then, similarly, (49′) and (51′) for V_{2e} and V_{2o}, we have

$$V_{1e} = \frac{R_3 V_1 + V_3}{1 + R_3} = V_{3e} \tag{64}$$

$$V_{1o} = \frac{V_1 - V_3}{1 + R_3} = -\frac{V_{3o}}{R_3} \tag{65}$$

$$V_{2e} = \frac{R_3 V_2 + V_4}{1 + R_3} = V_{4e} \tag{66}$$

$$V_{2o} = \frac{V_2 - V_4}{1 + R_3} = -\frac{V_{4o}}{R_3}. \tag{67}$$

Therefore, by substituting these four voltage components in the right-hand vectors of (54), (55), (58), and (59), the current components I_{1e}, I_{1o}, I_{2e}, and I_{2o} can be computed.

These current components can then be substituted in (44′)–(47′) yielding the expressions of the four port currents I_1, I_2, I_3, and I_4 as linear combinations of the four port voltages V_1, V_2, V_3, and V_4.

The 16 coefficients of these four expressions then represent the 16 entries of the 4×4 Y-matrix of the nonsymmetrical and asynchronous coupled-line 4-port. They respect the mutual identities dictated by end-to-end symmetry and by reciprocity between any two distinct ports, regardless of the specific values of the even-mode and odd-mode wave velocities v_e and v_o.

Six different entries need to be given to fully specify the 4×4 Y-matrix, and these are

$$Y_{11} = Y_{22} = -\frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^a R_3}{\tan \theta_e} + \frac{1}{Z_{0o}{}^a \tan \theta_o}\right) \tag{68}$$

$$Y_{33} = Y_{44} = -\frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^b}{\tan \theta_e} + \frac{R_3}{Z_{0o}{}^b \tan \theta_o}\right) \tag{69}$$

$$Y_{12} = Y_{21} = \frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^a R_3}{\sin \theta_e} + \frac{1}{Z_{0o}{}^a \sin \theta_o}\right) \tag{70}$$

$$Y_{34} = Y_{43} = \frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^b}{\sin \theta_e} + \frac{R_3}{Z_{0o}{}^b \sin \theta_o}\right) \tag{71}$$

$$\begin{aligned} Y_{13} = Y_{31} = Y_{24} = Y_{42} &= -\frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^a}{\tan \theta_e} - \frac{1}{Z_{0o}{}^a \tan \theta_o}\right) \\ &= -\frac{jR_3}{1 + R_3}\left(\frac{Y_{0e}{}^b}{\tan \theta_e} - \frac{1}{Z_{0o}{}^b \tan \theta_o}\right) \end{aligned} \tag{72}$$

$$\begin{aligned} Y_{14} = Y_{41} = Y_{23} = Y_{32} &= \frac{j}{1 + R_3}\left(\frac{Y_{0e}{}^a}{\sin \theta_e} - \frac{1}{Z_{0o}{}^a \sin \theta_o}\right) \\ &= \frac{jR_3}{1 + R_3}\left(\frac{Y_{0e}{}^b}{\sin \theta_e} - \frac{1}{Z_{0o}{}^b \sin \theta_o}\right). \end{aligned} \tag{73}$$

In a similar way, (54) and (55) could be rewritten through matrix inversion as

$$\begin{vmatrix} V_{1e} \\ \\ V_{2e} \end{vmatrix} = | Y_E{}^a |^{-1} \cdot \begin{vmatrix} I_{1e} \\ \\ I_{2e} \end{vmatrix} \tag{74}$$

$$\begin{vmatrix} V_{1e} \\ \\ V_{2e} \end{vmatrix} = | Y_E{}^b |^{-1} \cdot \begin{vmatrix} \frac{I_{1e}}{R_3} \\ \\ \frac{I_{2e}}{R_3} \end{vmatrix} \tag{75}$$

and directly combined with (56) and (57) to obtain the expression of the 4 × 4 Z-matrix of the nonsymmetrical and asynchronous coupled-line 4-port. The six different entries of the Z-matrix are given by

$$Z_{11} = Z_{22} = -\frac{j}{1+R_3}\left(\frac{R_3}{Y_{0e}{}^a \tan\theta_e} + \frac{Z_{0o}{}^a}{\tan\theta_o}\right) \tag{76}$$

$$Z_{33} = Z_{44} = -\frac{j}{1+R_3}\left(\frac{1}{Y_{0e}{}^b \tan\theta_e} + \frac{R_3 Z_{0o}{}^b}{\tan\theta_o}\right) \tag{77}$$

$$Z_{12} = Z_{21} = -\frac{j}{1+R_3}\left(\frac{R_3}{Y_{0e}{}^a \sin\theta_e} + \frac{Z_{0o}{}^a}{\sin\theta_o}\right) \tag{78}$$

$$Z_{34} = Z_{43} = -\frac{j}{1+R_3}\left(\frac{1}{Y_{0e}{}^b \sin\theta_e} + \frac{R_3 Z_{0o}{}^b}{\sin\theta_o}\right) \tag{79}$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = -\frac{jR_3}{1+R_3}\left(\frac{1}{Y_{0e}{}^a \tan\theta_e} - \frac{Z_{0o}{}^a}{\tan\theta_o}\right) = -\frac{j}{1+R_3}\left(\frac{1}{Y_{0e}{}^b \tan\theta_e} - \frac{Z_{0o}{}^b}{\tan\theta_o}\right) \tag{80}$$

$$Z_{14} = Z_{41} = Z_{23} = Z_{32} = -\frac{jR_3}{1+R_3}\left(\frac{1}{Y_{0e}{}^a \sin\theta_e} - \frac{Z_{0o}{}^a}{\sin\theta_e}\right) = -\frac{j}{1+R_3}\left(\frac{1}{Y_{0e}{}^b \sin\theta_e} - \frac{Z_{0o}{}^b}{\sin\theta_o}\right). \tag{81}$$

It is very easy to verify that the expressions (68)–(73) given for the entries of the Y-matrix and the expressions (76)–(81) given for the entries of the Z-matrix reduce to the corresponding expressions given for a homogeneous medium [3], [4] for $\theta_e = \theta_o = \theta$. Also they will reduce to the corresponding expressions of symmetrical lines for $R_3 = 1$ [1], [2].

Further the expressions (76)–(81) defining the 4 × 4 Z-matrix have been successfully tested against the values computed by numerically inverting the Y-matrix as defined by the expressions (68)–(73). This proves that the two matrices represent the same 4-port.

Finally, the closed-form expressions of the S-parameters of the nonsymmetrical, asynchronous 4-port can be derived by directly relating the four port voltages and the four port currents to the internal even- and odd-mode waves propagating along the two lines.

We assume here the wave propagation upon a pair of nonsymmetrical and asynchronous coupled lines a and b to be represented by a total of four internal waves, each wave having voltage and current components on either line.

Referring to line a these waves are:

1) The even-mode incident (forward) wave V_{iae} which is assumed to propagate from port 1 toward port 2;

2) The odd-mode incident (forward) wave V_{iao} also propagating from port 1 to port 2;

3) The even-mode reflected (backward) wave V_{rae} which is assumed to propagate from port 2 toward port 1;

4) The odd-mode reflected (backward) wave V_{rao} also propagating from port 2 to port 1 on line a.

The corresponding voltage waves on line b are V_{ibe}, V_{ibo}, V_{rbe}, and V_{rbo}. The incident (forward) waves are assumed to propagate from port 3 to port 4 and the reflected waves (backward waves) from port 4 to port 3.

The electrical length of the lines is assumed to be the same and expressed by θ_e for the even-mode waves and by θ_o for the odd-mode waves.

By assuming port 2 at the end of line a as the origin of a reference coordinate system with the positive semiaxis pointing toward port 1 and also as the reference point for the phases of all considered waves we can write the four port voltages V_1, V_2, V_3, V_4 as local additions of four voltage-wave terms

$$V_1 = V_{iae}\exp(j\theta_e) + V_{iao}\exp(j\theta_o) + V_{rae} \cdot \exp(-j\theta_e) + V_{rao}\exp(-j\theta_o) \tag{82}$$

$$V_2 = V_{iae} + V_{iao} + V_{rae} + V_{rao} \tag{83}$$

$$V_3 = V_{iae}\exp(j\theta_e) - R_3 V_{iao}\exp(j\theta_o) + V_{rae} \cdot \exp(-j\theta_e) - R_3 V_{rao}\exp(-j\theta_o) \tag{84}$$

$$V_4 = V_{iae} - R_3 V_{iao} + V_{rae} - R_3 V_{rao}. \tag{85}$$

Let us now consider the current waves associated with the already introduced voltage waves.

The even-mode currents can be expressed by multiplying the even-mode voltages by the appropriate even-mode line admittances $Y_{0e}{}^a$ or $Y_{0e}{}^b$. Similarly, the odd-mode currents can be expressed by dividing the odd-mode voltages by the appropriate odd-mode line impedance $Z_{0o}{}^a$ or $Z_{0o}{}^b$.

The four port currents I_1, I_2, I_3, I_4 may be expressed as local additions of four current-wave terms. But by expressing the wave currents in terms of wave voltages we obtain

$$I_1 = Y_{0e}{}^a V_{iae} \exp(j\theta_e) + \frac{V_{ia0}}{Z_{0o}{}^a} \exp(j\theta_o) - Y_{0e}{}^a V_{rae} \cdot \exp(-j\theta_e) - \frac{V_{rao}}{Z_{0o}{}^a} \exp(-j\theta_o) \quad (86)$$

$$I_2 = -Y_{0e}{}^a V_{iae} - \frac{V_{iao}}{Z_{0o}{}^a} + Y_{0e}{}^a V_{rae} + \frac{V_{rao}}{Z_{0o}{}^a} \quad (87)$$

$$I_3 = Y_{0e}{}^b V_{iae} \exp(j\theta_e) - R_3 \frac{V_{iao}}{Z_{0o}{}^b} \exp(j\theta_o) - Y_{0e}{}^b V_{rae} \cdot \exp(-j\theta_e) + R_3 \frac{V_{rao}}{Z_{0o}{}^b} \exp(-j\theta_o) \quad (88)$$

$$I_4 = -Y_{0e}{}^b V_{iae} + R_3 \frac{V_{iao}}{Z_{0o}{}^b} + Y_{0e}{}^b V_{rae} - R_3 \frac{V_{rao}}{Z_{0o}{}^b}. \quad (89)$$

Caution must be exercised in the choice of the 16 signs of the various current-wave terms. The rule, which follows from current-sign conventions, is that incident-wave currents are positive at ports 1 and 3, negative at ports 2 and 4. Conversely, reflected-wave currents are negative at ports 1 and 3 and positive at ports 2 and 4.

This rule is true if the corresponding wave voltage is positive, which is not the case for the odd-mode voltages of line b. Here we have sign reversal because the corresponding voltage is negative.

The two sets of equations (82)–(85) and (86)–(89) may both be solved for the four voltage waves V_{iae}, V_{iao}, V_{rae}, and V_{rao} so that each wave is expressed once as a function of the port voltages and once as a function of the port currents. At this point boundary conditions can be set specifying the values of the external impedances at the four ports and the position of one voltage source. For instance, by assuming a generator with EMF, E_1 and internal impedance Z_0 to be connected at port 1 and loads of impedance Z_0 to be connected at the remaining ports 2, 3, and 4, we have

$$I_1 = \frac{1}{Z_0}(E_1 - V_1) \quad (90)$$

$$I_2 = -\frac{V_2}{Z_0} \quad (91)$$

$$I_3 = -\frac{V_3}{Z_0} \quad (92)$$

$$I_4 = -\frac{V_4}{Z_0}. \quad (93)$$

By substituting these expressions of the port currents in the expressions of the wave voltages as functions of the same currents as obtained from (86)–(89), new expressions of these wave voltages are obtained which result again in being functions of the four port voltages. By equating this new set of four expressions to the corresponding solutions obtained from the set (82)–(85), the wave voltages can be eliminated and the four port voltages V_1, V_2, V_3, and V_4 can be directly expressed as functions of the system parameters and of the EMF, E_1. These operations actually provide the voltage transfer functions relating the four port voltages to the source EMF, E_1. These four transfer functions are directly related to the entries S_{11}, S_{12}, S_{13}, and S_{14} of the first row of 4×4 S-parameter matrix, which are all different from one another. By moving the generator from port 1 to port 3 the four entries of the third row of the 4×4 S matrix can be obtained. These will provide the two remaining entries S_{33} and S_{34} necessary to completely specify the 4×4 S-matrix and in addition confirm the equality of S_{31} to S_{13} and S_{32} to S_{14}, as required by reciprocity. It is of course possible to move the generator to ports 2 and 4 but this would yield the entries of the second and, respectively, the fourth row of the S-parameter matrix which, because of the peculiar matrix structure, would provide no additional new information.

The six different entries defining the 4×4 S-parameter matrix obtained through the outlined procedure are given by

$$S_{11} = S_{22} = \frac{N_{1R} + jN_{1I}}{D_R + jD_I} \quad (94)$$

$$S_{33} = S_{44} = \frac{N_{2R} + jN_{2I}}{D_R + jD_I} \quad (95)$$

$$S_{12} = S_{21} = \frac{N_{3R} + jN_{3I}}{D_R + jD_I} \quad (96)$$

$$S_{34} = S_{43} = \frac{N_{4R} + jN_{4I}}{D_R + jD_I} \quad (97)$$

$$S_{13} = S_{31} = S_{24} = S_{42} = \frac{N_{5R} + jN_{5I}}{D_R + jD_I} \quad (98)$$

$$S_{14} = S_{41} = S_{23} = S_{32} = \frac{N_{6R} + jN_{6I}}{D_R + jD_I} \quad (99)$$

where

$$D_R = 2[(1 - R_3)^2 - (3 + 2R_3 + 3R_3^2) \cos\theta_e \cos\theta_o] + \left[(1 + R_3)^2 \left(R_3 \frac{z_{0o}{}^a}{y_{0e}{}^a} + \frac{1}{R_3} \frac{y_{0e}{}^a}{z_{0o}{}^a}\right) + \frac{4R_3}{y_{0e}{}^a z_{0o}{}^a} + \frac{(1 + R_3^2)^2}{R_3} y_{0e}{}^a z_{0o}{}^a\right] \sin\theta_e \sin\theta_o \quad (100)$$

$$D_I = -2(1 + R_3) \left\{\left[(1 + R_3^2) z_{0o}{}^a + \frac{2}{z_{0o}{}^a}\right] \sin\theta_o \cos\theta_e + \left[(1 + R_3^2) \frac{y_{0e}{}^a}{R_3} + 2 \frac{R_3}{y_{0e}{}^a}\right] \sin\theta_e \cos\theta_o\right\} \quad (101)$$

$$N_{1R} = 2(1 - R_3^2)(1 - \cos\theta_e \cos\theta_o) + \left[(1+R_3)^2\left(R_3 \frac{z_{0o}^a}{y_{0e}^a} - \frac{1}{R_3}\frac{y_{0e}^a}{z_{0o}^a}\right) + \frac{1-R_3^4}{R_3} y_{0e}^a z_{0o}^a\right] \sin\theta_e \sin\theta_o \tag{102}$$

$$N_{1I} = -2(1+R_3)\left[\left(z_{0o}^a - \frac{1}{z_{0o}^a}\right)\sin\theta_o \cos\theta_e - \left(y_{0e}^a - \frac{1}{y_{0e}^a}\right) R_3 \sin\theta_e \cos\theta_o\right] \tag{103}$$

$$N_{2R} = -2(1 - R_3^2)(1 - \cos\theta_e \cos\theta_o) + \left[(1+R_3)^2\left(R_3 \frac{z_{0o}^a}{y_{0e}^a} - \frac{1}{R_3}\frac{y_{0e}^a}{z_{0o}^a}\right) - \frac{1-R_3^4}{R_3} y_{0e}^a z_{0o}^a\right] \sin\theta_e \sin\theta_o \tag{104}$$

$$N_{2I} = -2(1+R_3)\left[\left(R_3^2 z_{0o}^a - \frac{1}{z_{0o}^a}\right)\sin\theta_o \cos\theta_e + \left(\frac{R_3}{y_{0e}^a} - \frac{y_{0e}^a}{R_3}\right)\sin\theta_e \cos\theta_o\right] \tag{105}$$

$$N_{3R} = -4(1+R_3)(\cos\theta_e + R_3 \cos\theta_o) \tag{106}$$

$$N_{3I} = -2(1+R_3)\left[\left(\frac{R_3}{y_{0e}^a} + \frac{y_{0e}^a}{R_3}\right)\sin\theta_e + \left(R_3^2 z_{0o}^a + \frac{1}{z_{0o}^a}\right)\sin\theta_o\right] \tag{107}$$

$$N_{4R} = -4(1+R_3)(R_3 \cos\theta_e + \cos\theta_o) \tag{108}$$

$$N_{4I} = -2(1+R_3)\left[\left(y_{0e}^a + \frac{1}{y_{0e}^a}\right)R_3 \sin\theta_e + \left(z_{0o}^a + \frac{1}{z_{0o}^a}\right)\sin\theta_o\right] \tag{109}$$

$$N_{5R} = 2(1 - R_3)^2(1 - \cos\theta_e \cos\theta_o) - 2\left[(1+R_3^2) y_{0e}^a z_{0o}^a - \frac{2R_3}{y_{0e}^a z_{0o}^a}\right] \cdot \sin\theta_e \sin\theta_o \tag{110}$$

$$N_{5I} = 2(1+R_3)\left[\left(R_3 z_{0o}^a - \frac{1}{z_{0o}^a}\right)\sin\theta_o \cos\theta_e - \left(\frac{R_3}{y_{0e}^a} - y_{0e}^a\right)\sin\theta_e \cos\theta_o\right] \tag{111}$$

$$N_{6R} = 2(1+R_3)^2(\cos\theta_e - \cos\theta_o) \tag{112}$$

$$N_{6I} = 2(1+R_3)\left[\left(\frac{R_3}{y_{0e}^a} + y_{0e}^a\right)\sin\theta_e - \left(R_3 z_{0o}^a + \frac{1}{z_{0o}^a}\right)\sin\theta_o\right] \tag{113}$$

where y_{0e}^a and z_{0o}^a are the normalized parameters of line a.

The S-parameter matrix as defined by the previously given closed-form expressions, has been tested against the results of numerical computations of the same parameters based on the general formula

$$|S| = \{|U| + |y|\}^{-1}\{|U| - |y|\} \tag{114}$$

where $|U|$ represents the 4×4 identity matrix and $|y|$ is the normalized 4×4 Y-matrix obtained from the expressions (68)–(73) by multiplying all the entries by the external impedance Z_0 [14]. Recently, more general expressions of the six entries of the 4×4 S-parameter matrix have also been derived for a situation in which the value of the termination Z_0^a at either end of line a (ports 1 and 2) is different than the value of the termination Z_0^b at either end of line b (ports 3 and 4) [15]. These expressions led to the design of asynchronous codirectonal couplers with large impedance ratios between the two lines and almost total coupling, characterized by absolutely linear phase rotation [16].

Also alternative closed-form expressions of the S-parameters have been obtained by formally carrying out the operations of matrix inversion and multiplication specified by (114) [17].[2] These alternative expressions have also been tested numerically for correctness.

V. EXPERIMENTAL RESULTS

Two basic experiments were performed in our laboratory by using coupled-line test fixtures, such as the one in Fig. 2, having parallel ground planes with 0.5-in spacing and suspended ceramic substrate orthogonal to the ground planes. The lines are broadside-coupled gold runs of unequal width, and the cross-section geometry is that shown in Fig. 1. A 3-in and a 16-in structure were used.

In the first experiment [Fig. 3(a) and (b)] a voltage even mode was launched down the lines by connecting both input ports 1 and 3 (Fig. 1) in parallel. A 25-ps-rise-time tunnel-diode step generator was used and 12-GHz samplers were connected at the output ports 2 and 4. The input even-mode step waveform was found to propagate along both coupled lines at the same velocity, slightly lower than the velocity of light in vacuum regardless of line asymmetry (0.065-in and 0.0135-in strip widths were used). The output step signals on the 50-Ω output lines appear unequal in magnitude because of the unequal reflections at the two output transitions

[2] Credit is due to Dr. N. R. Franzen, of our department, for developing a method of formal matrix inversion applicable to 4 × 4 block matrices having the symmetry pattern of the given Y-, Z-, and S-matrices.

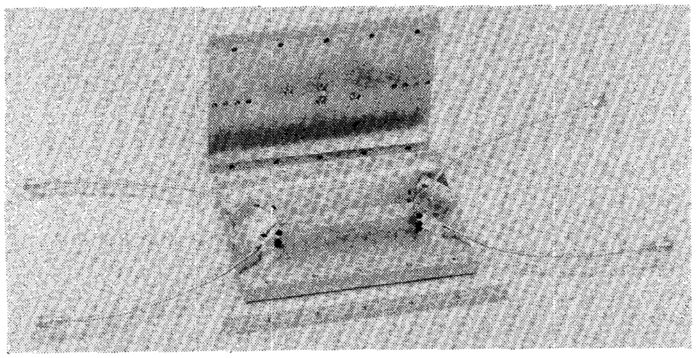
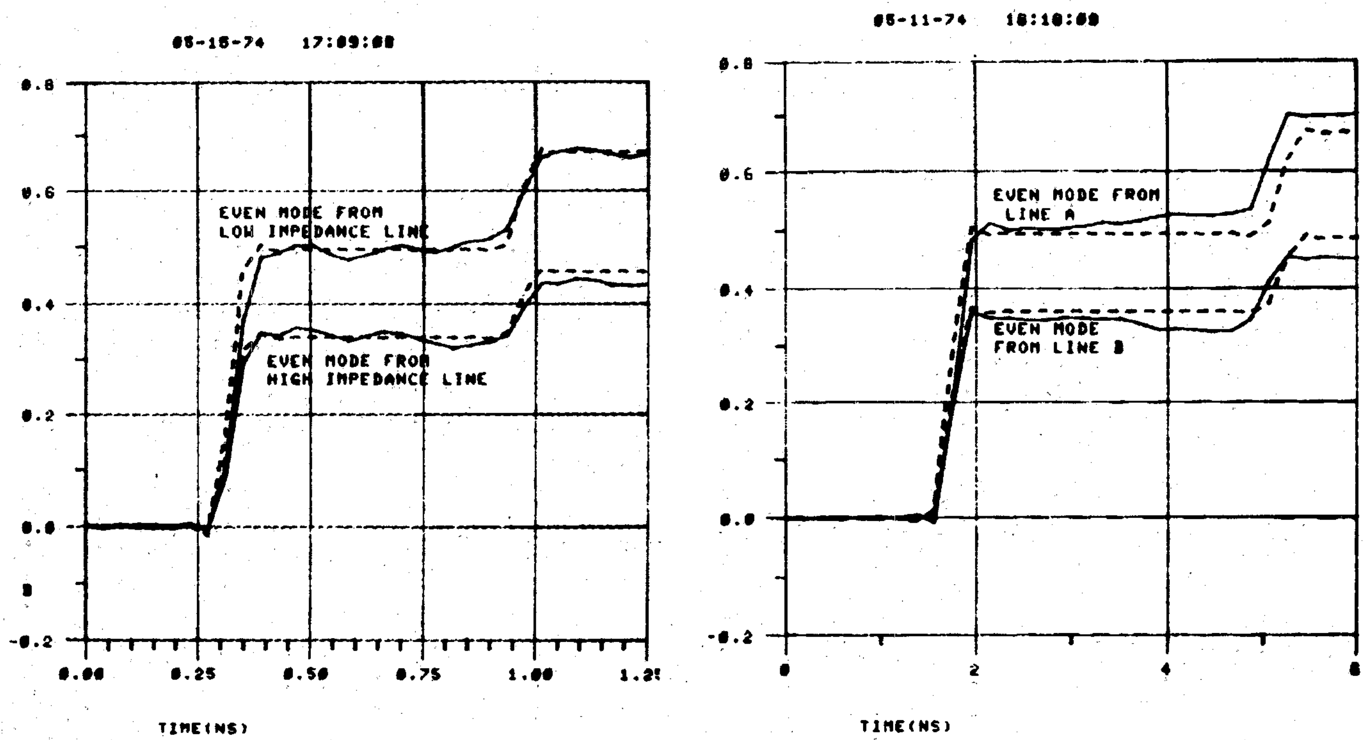


Fig. 2. Test fixture for even- and odd-mode wave propagation experiments upon nonsymmetrical asynchronous parallel coupled lines. A 4-port launcher arrangement is shown on the 3-in-long fixture.

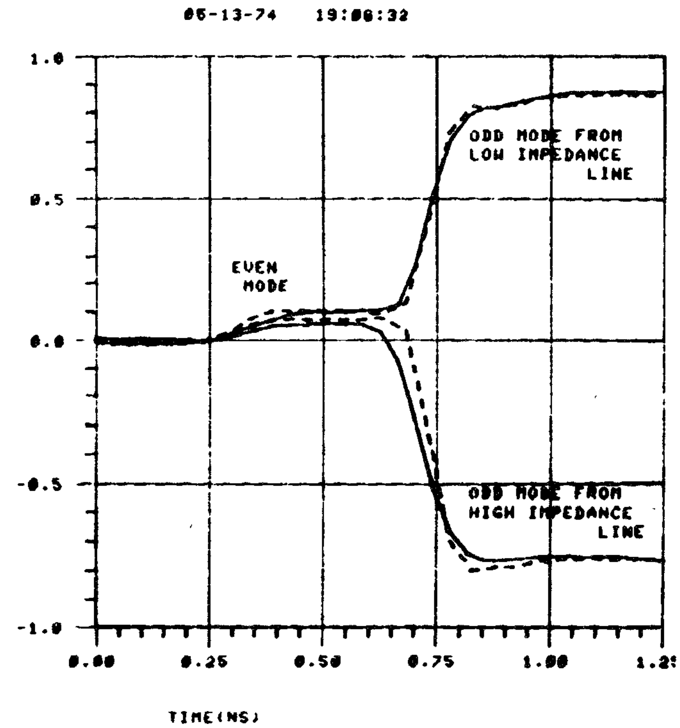


(a) (b)

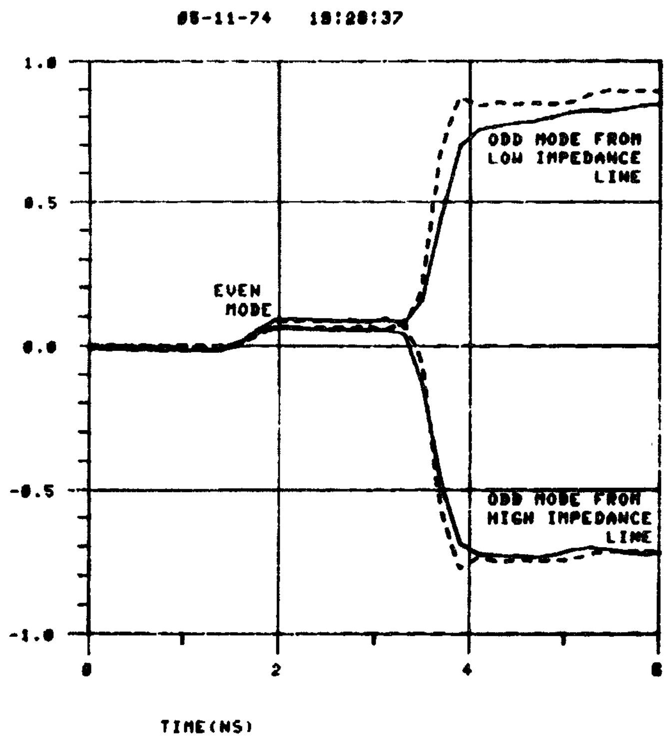
Fig. 3. Even-mode step-wave propagation experiment. The two equal 25-ps-rise-time input steps are seen to reappear synchronously at the low-impedance and the high-impedance output ports 2 and 4. (a) Results from 3-in-long fixture. (b) Results from 16-in-long fixture. Ports 1 and 3 of lines a and b are shorted to one another. The dashed lines are theoretical step responses computed from frequency-domain S-parameter data through FFT.

In a second experiment [Fig. 4(a) and (b)], a push–pull 25-ps-rise-time tunnel-diode step generator was connected to the input lines and adjusted for step synchronism as an attempt at launching an odd mode down the lines. Obtaining a pure odd mode is difficult because of the unequal reflections at the two input transitions. However, it was considered irrelevant because the different velocities of the two modes separate them in time at the output. The results obtained proved that the incident, push–pull step excitation applied at the line inputs breaks down along the line length into two nonsynchronous components consisting of a faster voltage even mode having the polarity of the signal at the wide-line input, and a much slower current odd mode, having the largest voltage across the narrow line.

The values of the line parameters Y_{0e}^{a}, Y_{0e}^{b}, Z_{0o}^{a}, and Z_{0o}^{b} may be computed from step-amplitude values obtained in the two experiments and have been used, with the experimental values of the even-mode and odd-mode velocities to compute the theoretical transient responses shown as dashed lines in Figs. 3 and 4. This computation was performed by converting frequency-domain S-param-



(a)



(b)

Fig. 4. Push–pull step-excitation experiment. The 25-ps-rise-time input steps having opposite polarity are seen to break down to a faster even-mode component having the polarity of the step at the input of the low-impedance line [positive step on the low-impedance line in Fig. 4(a) and (b)] and two slower odd-mode components having unequal voltage magnitudes, the larger voltage appearing across the higher impedance line. (a) Results from 3-in-long fixture. (b) Results from 16-in-long fixture.

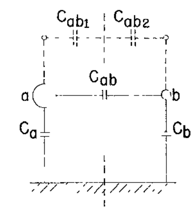


Fig. 5. A nonsymmetrical pair of parallel-coupled lines: C_a and C_b are line-to-ground capacitances per unit length, C_{ab} is the line-to-line capacitance per unit length which must be considered, for the odd mode, as the series combination of the two unequal capacitances C_{ab1} and C_{ab2} ($C_{ab1}/C_a = C_{ab2}/C_b$).

eter values to the time domain through a fast Fourier transform (FFT).

VI. CONCLUSION

In a large class of nonsymmetrical coupled lines with nonhomogeneous propagation medium, a voltage even mode and a current odd mode are found to be the fundamental uncoupled modes of the structure.

The two modes are characterized by having, respectively, wave voltages of equal magnitude and phase, and wave currents of equal magnitude and opposite phase.

This causes the even-mode currents on the two lines to be in the mutual ratio of the per-unit-length line-to-ground capacitances and the odd-mode voltages to be in the inverse ratio of these capacitances.

The condition for the existence of these simple modes, called congruence condition, is $C_a/C_b = (L_b - M)/(L_a - M)$.

This condition is satisfied by nonsymmetrical lines in a homogeneous medium and implies, for a nonhomogeneous dielectric, that the ratio of the two line-to-ground capacitances C_a and C_b is the same in the filled and in the empty structure (see Fig. 5).

The existence of these new modes has been proved experimentally in suspended-substrate broadside-coupled striplines.

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Annotated Literature Survey of Microwave Ferrite Control Components and Materials for 1968–1974

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***Abstract*—An annotated literature survey covering major development in the area of microwave ferrite control components and material primarily for the 1968–1974 period is presented.**

I-1. INTRODUCTION

As PART of the responsibility for monitoring the field of microwave ferrite control components and materials the IEEE Microwave Theory and Techniques Society Ferrite Technical Committee (MTT-13) and the IEEE Magnetics Society's Technical Committee on High Frequency Materials have combined their resources to conduct a survey of the most significant papers that have appeared since the publication of the book by Lax and Button [I-1-1] in 1962, the 1968 survey article by Soohoo [I-1-2], and the 1969 book by Fay and Von Aulock [I-1-3]. The time for a new survey appears particularly appropriate in that this field has seen major advances in ferrite components and in materials. The field has matured and has reached a high level of sophistication in component optimization and in materials technology.

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The emphasis in this article is on the ferrimagnetic control component and systems utilization of such components, although material aspects are considered also. A companion survey paper [I-1-4] has been published in the May issue of the IEEE TRANSACTIONS ON MAGNETICS. The article published therein has greater emphasis on materials and includes a section by C. E. Patton on the loss mechanisms in ferrimagnetic materials. A further companion paper by Knerr [I-1-5] covers the English language literature on circulators and isolators.

The survey was organized and pursued at joint meetings of the two technical committees with members of these committees accepting responsibility for the major topics. The section on nonlinear ferrite devices and filters was prepared by J. L. Allen, the sections on phase shifters and integrated circuits were contributed by L. R. Whicker, C. R. Boyd, Jr., R. Tang, and R. G. Roberts. P. de Santis and D. M. Bolle collaborated on the section on the edge-guided mode, and R. G. West and L. K. Wilson prepared the section on characterization and material properties of ferrites. The reviews of the foreign literatures were conducted by J. Nicolas and A. Priou (Materials/Devices,